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NONLINEAR ANALYSIS OF SHELLS OF REVOLUTION BY THE MATRIX DISPLACEMENT METHOD

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Objective

The objective of this paper is to present a method for the nonlinear (large deflection) analysis of shells of revolution under both symmetrical and asymmetrical loadings. Theoretical results are compared with experimental results.

Introduction

The matrix displacement method of structural analysis has been proved to be quite accurate for linear[1-3] and stability[4] analysis of shells of revolution. The method is easy to apply and yields good results when a relatively small number of elements are used. This latter point makes the matrix displacement method ideally suited for the nonlinear analysis of shells of revolution and is the reason it was selected for this study.

Method of Approach

Assumptions

The following assumptions were made in the study:

1. Novozhilov's[5] nonlinear thin shell theory applies.
2. The linear strains are small compared with the rotations.
3. The linear strains and rotations may be approximated over the meridional length of the element as their values at the middle of the element.

Basic Equations

In the analysis of shells of revolution by the matrix displacement method the displacements are represented by Fourier series in the circumferential direction and polynomials in the meridional direction:

$$U = \sum (\alpha_5^i + \alpha_6^i s) \cos i\theta + \sum (\bar{\alpha}_5^i + \bar{\alpha}_6^i s) \sin i\theta$$

$$V = \sum (\alpha_7^i + \alpha_8^i s) \sin i\theta + \sum (\bar{\alpha}_7^i + \bar{\alpha}_8^i s) \cos i\theta \qquad (1)$$

$$W = \sum (\alpha_1^i + \alpha_2^i s + \alpha_3^i s^2 + \alpha_4^i s^3) \cos i\theta$$
$$+ \sum (\bar{\alpha}_1^i + \bar{\alpha}_2^i s + \bar{\alpha}_3^i s^2 + \bar{\alpha}_4^i s^3) \sin i\theta$$

where U, V, and W are the deflections in the meridional, circumferential, and normal directions respectively.

α_i^j = generalized coordinates

S = meridional distance along element

θ = circumferential angle

From assumption 2 the expressions for the changes in curvatures are given by linear theory and the midsurface strains are given by

$$\epsilon_{11} = e_{11} + \frac{1}{2} e_{13}^2$$

$$\epsilon_{22} = e_{22} + \frac{1}{2} e_{23}^2 \qquad (2)$$

$$\epsilon_{12} = e_{12} + e_{21} + e_{13} e_{23}$$

where ϵ = total midsurface strain.

The expressions for the e's are given on page 190 of Ref. 5.

For this study the internal energy expression is separated into two parts:

$$U = U_1 + U_2 \qquad (3)$$

where

U = total internal energy

U_1 = expression for internal energy based on linear theory

U_2 = internal energy due to nonlinearities

As viewed here the matrix displacement method is an application of Castigliano's theorem

$$\frac{\partial U_1}{\partial q_i} = Q_i - \frac{\partial U_2}{\partial q_i} \tag{4}$$

where

q_i = generalized nodal coordinate (related to α's in Eq. 1)

Q_i = generalized forces

Equations of Equilibrium

Equation 4, in conjunction with Eqs. 1 and 2, yields the equations of equilibrium for any harmonic in the form

$$[K]\{q\} = \{Q\} - \left\{\frac{\partial U_2}{\partial q_i}\right\} \tag{5}$$

Nonlinear Terms

The treatment of the nonlinear terms, the experimental data, and the applications constitute the new and original contributions of this paper.

To evaluate the generalized forces due to nonlinearities in the strain displacement relations two approaches were used. The first approach did not use assumption 3. In this approach the assumed displacement functions were substituted into the strain energy expression for U_2 and the integration performed over the meridional and circumferential lengths of the element. The partial derivative of U_2 with respect to the generalized coordinate q_i was evaluated through chain rule differentiation with the generalized coordinate α_i serving as an intermediate variable. The partial derivative with respect to α_i was evaluated by summing over all harmonics (20 maximum). This was accomplished by using the displacement at the previous loading (load increment method) or the values at a previous solution (iteration method). After evaluating the loading due to nonlinearities for each element, the element loadings were combined to yield the total generalized forces.

The preceding method is long and requires considerable computer storage for the expressions for the nonlinear terms. Consequently, various approximations were tested numerically in an attempt to reduce the large number of terms due to nonlinearities.

It was found that the variation of the strains and rotations over the meridional length of the element may be approximated extremely well as the values at the middle of the element. Consequently, this approximation was incorporated into the analysis. The analysis still involves integrals around the circumference of the shell. However, these integrals are higher order combinations of sine and cosine terms and may be evaluated exactly.

Method of Solution

Equation 5 is solved by using both the incremental load approach and the iterational solution. In the incremental load approach the deflections at the previous loading or their extrapolated values are used to evaluate the nonlinear terms in Eq. 5. To date, the iterational solution has been applied only at the maximum loading, using the incremental load approach up to the last loading. The linear equations obtained from Eq. 5 are solved by Gaussian elimination.

Applications

Figures 1 and 2 present the circumferential and meridional stresses on the inside and outside surfaces of a pressure vessel under an internal loading of 50 psi. The geometry of the pressure vessel is shown on the figures. The material properties are Young's Modulus = 10×10^6 psi and Poisson's ratio = 0.33.

The linear analysis was conducted using three independent numerical programs. These included the finite difference procedure as well as the matrix displacement method. All three linear solutions agreed. The nonlinear analysis was conducted using 50 elements and a single harmonic. Ten load increments of 5 psi were used in the analysis.

The experimental data were obtained using a total of two hundred strain gages mounted on the inside and outside of the shell. A hydraulic pump was used to apply the internal pressure.

It is noted that the nonlinear analysis yields results which agree quite well with experimental data and agrees much better than the results from the linear analysis. The lack of complete agreement is probably due to the small radius to thickness ratio (10) of the toroidal section.

Figures 3 and 4 present a comparison of linear and nonlinear results for a spherical cap under a localized loading. The load was applied over a circle with a 2" radius with the origin 3° from the apex. The solution was obtained using 25 elements and 5 harmonics. Ten load increments of 10 psi each were used in the incremental load solution. After the loading reached its maximum value of 100 psi the iteration approach was used until convergence occurred.

It is noted that the linear solution underestimates the stresses in the shell. Furthermore, the incremental load approach, using ten increments, has not converged to the correct nonlinear solution obtained by iteration. However, the disagreement between the incremental load solution and the correct solution is much less than the difference between the linear and nonlinear solutions.

Conclusions

This is the first development for the nonlinear analysis of shells of revolution under arbitrary loadings. The efficiency and accuracy of the resulting numerical program make it of practical as well as theoretical value. This research has not been presented elsewhere.

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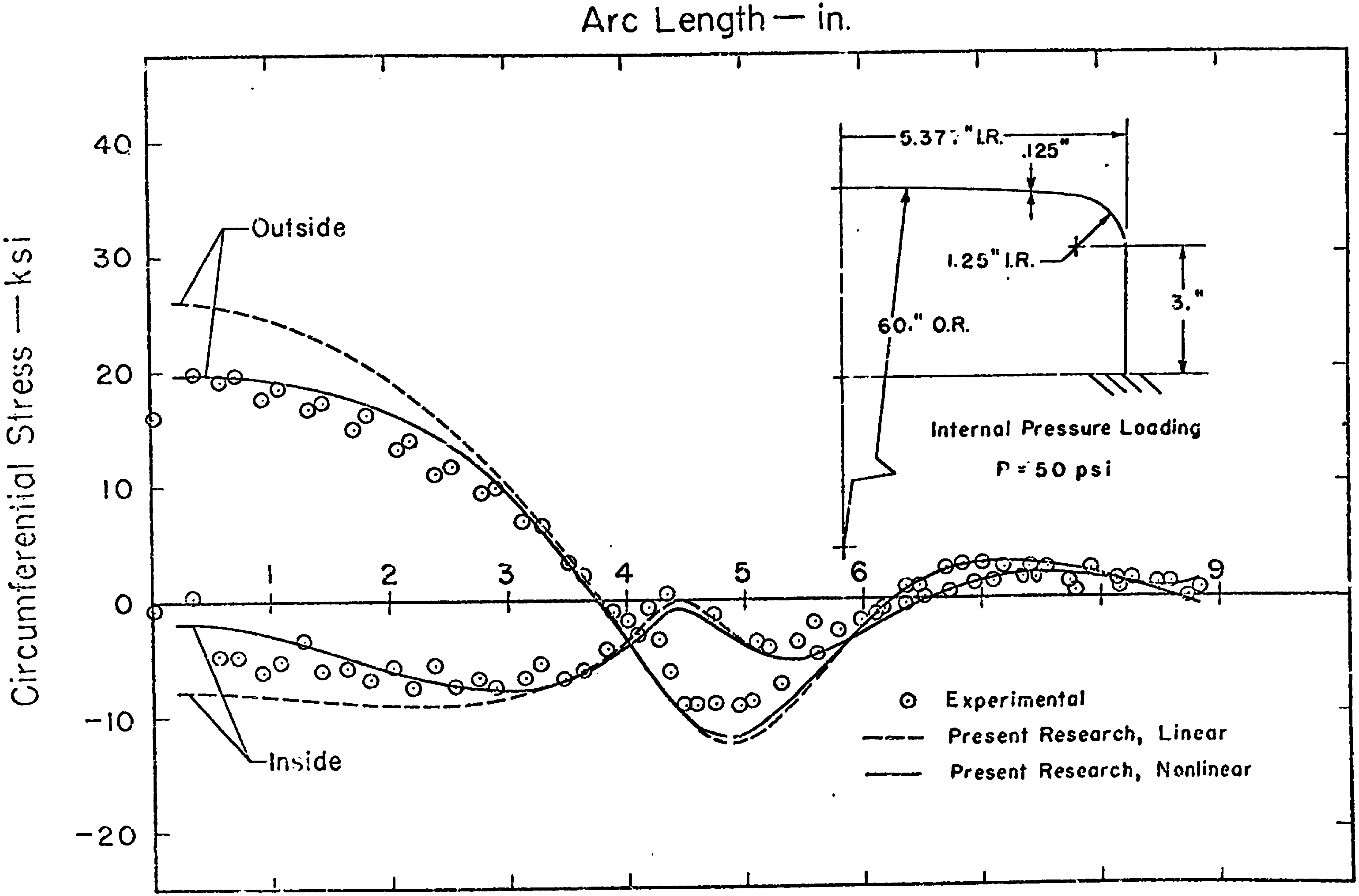

Fig. I CIRCUMFERENTIAL STRESS ALONG ARC LENGTH

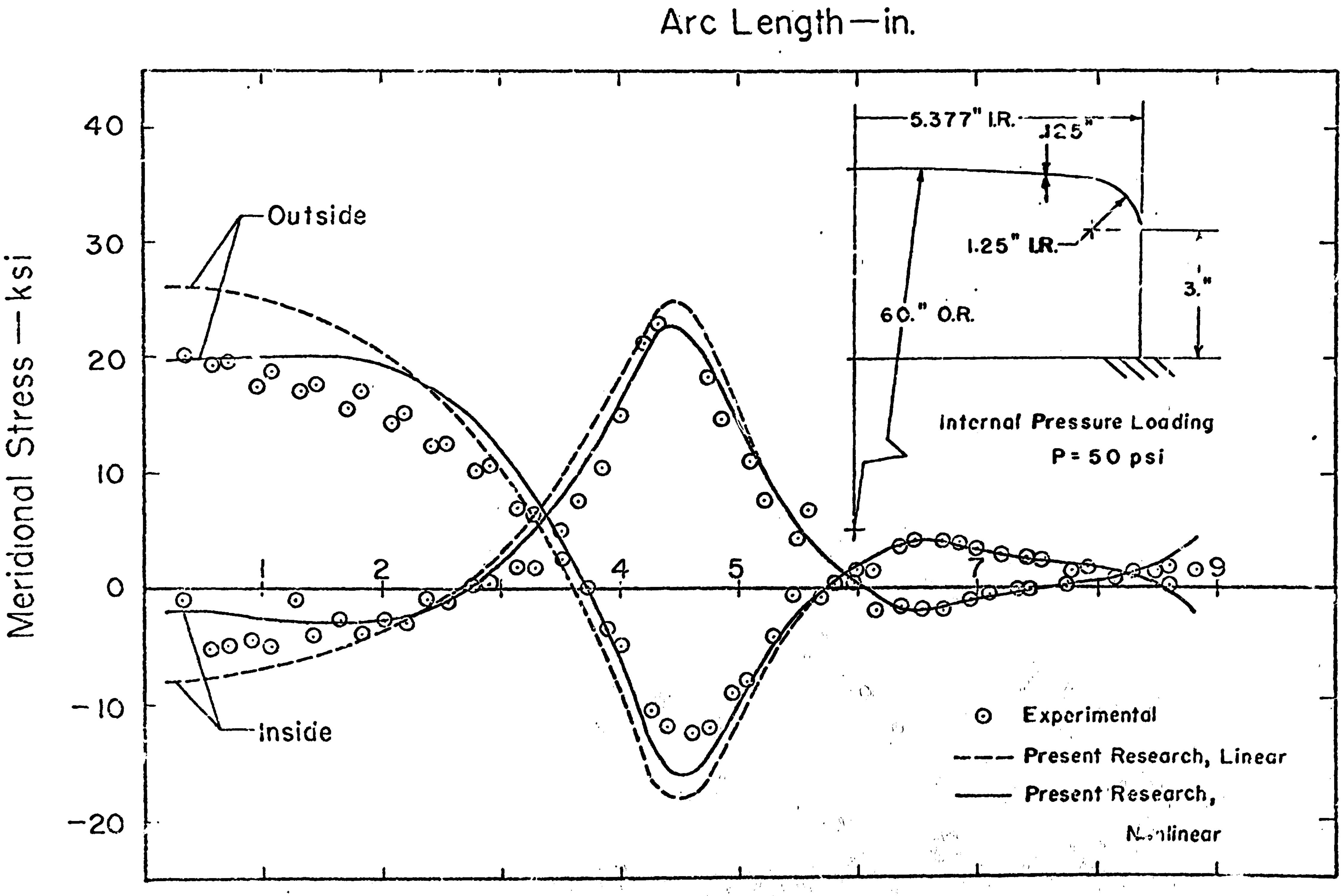

Fig. 2 MERIDIONAL STRESS ALONG ARC LENGTH

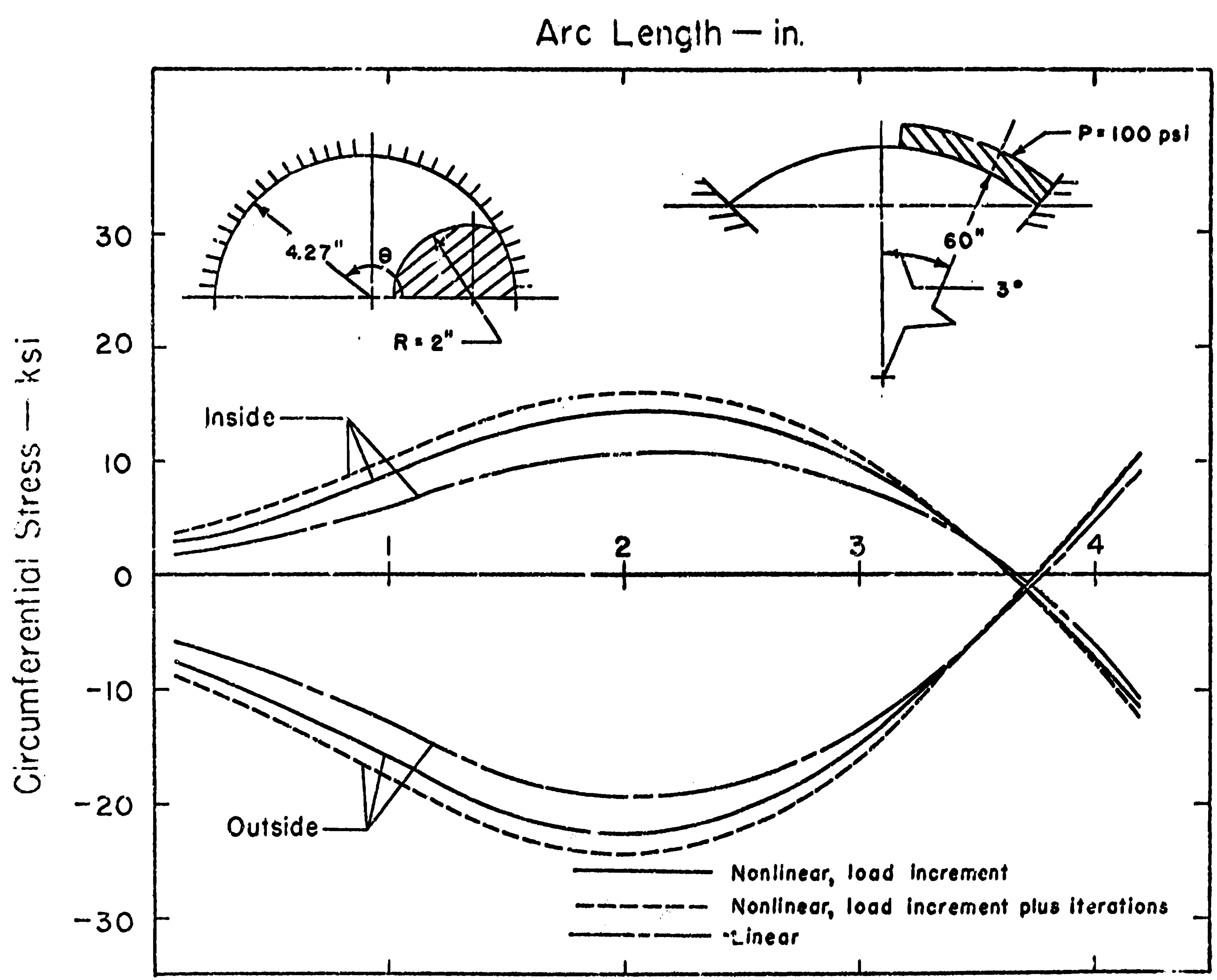

Fig. 3 CIRCUMFERENTIAL STRESS ALONG ARC LENGTH

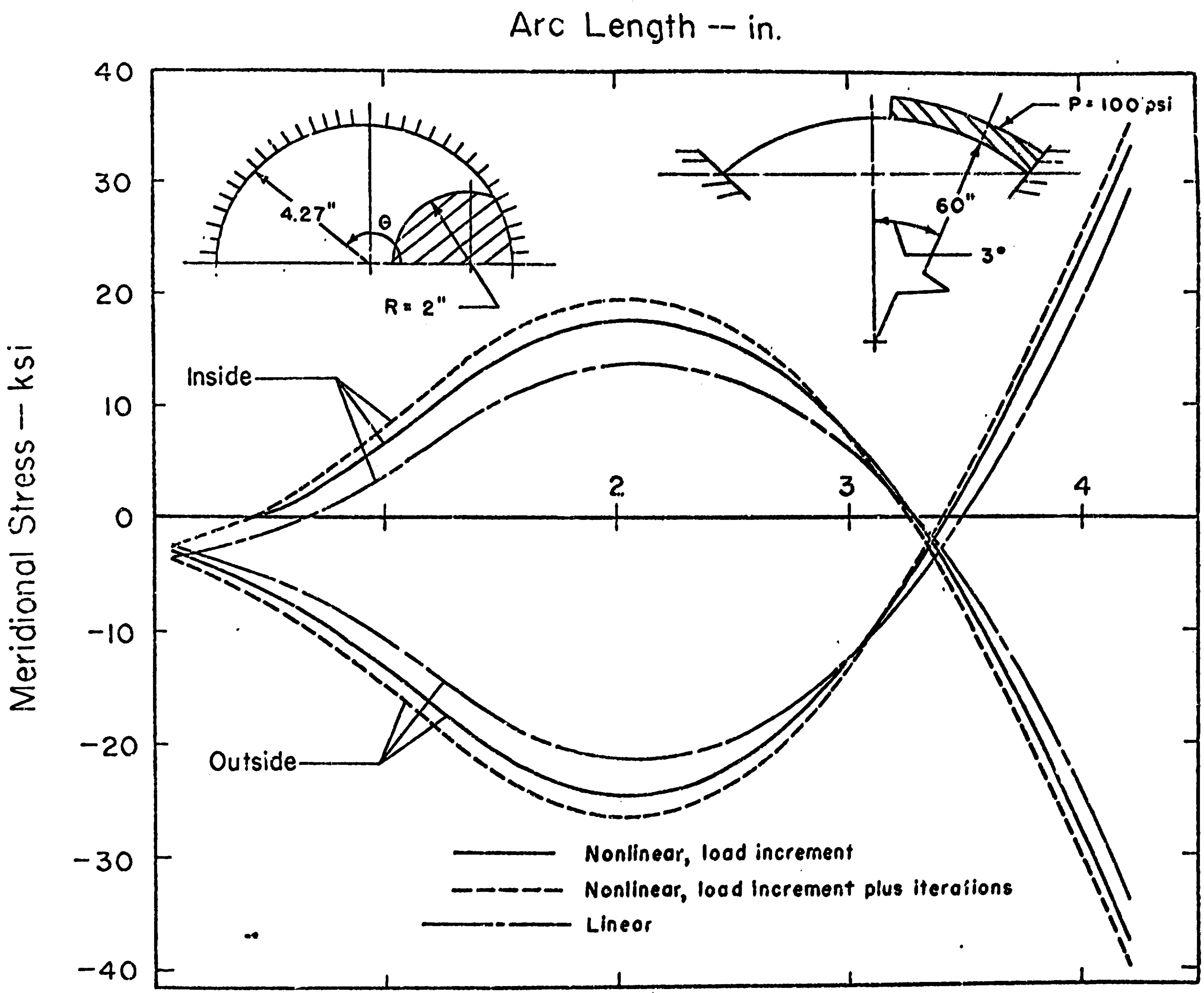

Fig. 4 MERIDIONAL STRESS ALONG ARC LENGTH